

Important Theorems

Continuity The function f is continuous at $x = c$ if $\lim_{x \rightarrow c} f(x) = f(c)$.

Absolute Extrema If f is continuous on the closed interval $[a, b]$ then f has an absolute maximum and an absolute minimum value. Furthermore, the absolute extrema will occur at either the critical points or the endpoints.

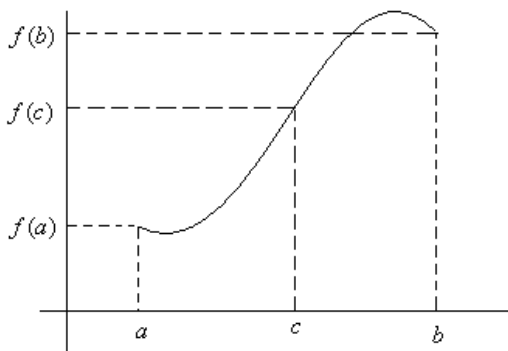
First Fundamental Theorem If f is continuous on $[a, b]$ and if F is any antiderivative for f , then $\int_a^b f(x)dx = F(b) - F(a)$.

Second Fundamental Theorem

a) If f is continuous on $[a, b]$ and if $F(x) = \int_a^x f(t)dt$, then $F'(x) = f(x)$.

b) If f is continuous on $[a, b]$ and if $F(x) = \int_a^{g(x)} f(t)dt$, then $F'(x) = f(g(x))g'(x)$.

Intermediate Value Theorem If f is a continuous function on the closed interval $[a, b]$, then f takes on every value between $f(a)$ and $f(b)$. In other words, if $f(a) \leq y_0 \leq f(b)$ then there exists a value c , $a \leq c \leq b$, such that $f(c) = y_0$.



Mean Value Theorem If f is continuous at every point in the closed interval $[a, b]$ and differentiable at every point of its interior (a, b) , then there is at least one point c in (a, b)

at which $f'(c) = \frac{f(b) - f(a)}{b - a}$.

Derivative of an Inverse Function Let f be a function that is differentiable on an interval I . If f possesses an inverse function g , then g is differentiable at any x for which $f'(g(x)) \neq 0$.

$g'(x) = \frac{1}{f'(g(x))}$. If we use the terminology f^{-1} instead of g we get:

$$(f^{-1})'(x) = \frac{1}{f'(f^{-1}(x))}.$$

L'Hopital's Rule If $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{0}{0}$ or $\frac{\infty}{\infty}$, then $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \lim_{x \rightarrow c} \frac{f'(x)}{g'(x)}$.

Trapezoid Rule $\int_a^b f(x) dx \approx \frac{b-a}{2n} [y_0 + 2y_1 + 2y_2 + \dots + 2y_{n-1} + y_n]$

Simpson's Rule $\int_a^b f(x) dx \approx \frac{b-a}{3n} [y_0 + 4y_1 + 2y_2 + 4y_3 + \dots + 4y_{n-1} + y_n]$