

CALCULUS REVIEW

Derivatives

$$\text{Product Rule: } (fg)' = f'g + fg'$$

$$(x^3 \sin x)' = 3x^2 \sin x + x^3 \cos x$$

$$\text{Quotient Rule: } \left(\frac{Hi}{Ho} \right)' = \frac{Ho DHi - Hi DHo}{HoHo}$$

$$\left(\frac{\sin x}{x^3} \right)' = \frac{x^3 \cos x - 3x^2 \sin x}{x^6}$$

$$\text{Chain Rule: } (f \circ g(x))' = f'(g(x))g'(x)$$

$$(\sin(x^3))' = 3x^2 \cos(x^3)$$

$$\text{Power Rule: } (f^n(x))' = n(f^{n-1}(x))f'(x)$$

$$(\sin^3 x)' = 3 \sin^2 x \cos x$$

Trig Functions

$$(\sin x)' = \cos x$$

$$(\sec x)' = \sec x \tan x$$

$$(\tan x)' = \sec^2 x$$

$$(\cos x)' = -\sin x$$

$$(\csc x)' = -\csc x \cot x$$

$$(\cot x)' = -\csc^2 x$$

Inverse Trig Functions

$$\frac{d}{dx}(\sin^{-1} u) = \frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}(\sec^{-1} u) = \frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}(\tan^{-1} u) = \frac{u'}{u^2+1}$$

$$\frac{d}{dx}(\cos^{-1} u) = -\frac{u'}{\sqrt{1-u^2}}$$

$$\frac{d}{dx}(\csc^{-1} u) = -\frac{u'}{|u|\sqrt{u^2-1}}$$

$$\frac{d}{dx}(\cot^{-1} u) = -\frac{u'}{u^2+1}$$

$$\text{Note: } \frac{d}{dx}(\tan^{-1} x) = \frac{1}{x^2+1}$$

Logs and Exponentials

$$(Ln u)' = \frac{u'}{u}$$

$$(Ln(x^2 + 1))' = \frac{2x}{x^2 + 1}$$

$$(e^{u(x)})' = u'(x)e^{u(x)}$$

$$(e^{x^3})' = 3x^2 e^{x^3}$$

CRITICAL POINTS are points, c , such that $f'(c) = 0$ or $f'(c)$ is undefined.

Relative extrema always occur at critical points. Not all critical points are relative extrema. To classify critical points use the **FIRST DERIVATIVE TEST** (number line).

Absolute extrema occur at critical points and endpoints. If there are no endpoints, you must show relative extrema are absolute extrema. This will always be the case if there is only one critical point.

$x(t)$, $y(t)$ and $s(t)$ are all common notations for position functions.

$$v(t) = \text{velocity} \quad v(t) = x'(t)$$

$$a(t) = \text{acceleration} \quad a(t) = v'(t) = x''(t)$$

Objects move to the right when $v(t) > 0$ (use a number line)

Integrals

$$\int_a^b f(x) dx = \int_a^c f(x) dx + \int_c^b f(x) dx \quad \text{where } a < c < b$$

$$\int_a^b cf(x) dx = c \int_a^b f(x) dx \qquad \int_a^b (f(x) \pm g(x)) dx = \int_a^b f(x) dx \pm \int_a^b g(x) dx$$

[Indefinite integrals may be used here instead of the definite integrals]

Integration by Parts formula: $\int u dv = uv - \int v du$

Note : there are no product/quotient rules for integrals!

$$\int x^n dx = \frac{x^{n+1}}{n+1} + C \quad \text{for } n \neq -1 \qquad \int \frac{1}{x} dx = \int \frac{dx}{x} = \text{Ln}x + C$$

Trig Functions

$$\int \sin x dx = -\cos x + C \qquad \int \sec x dx = \text{Ln}(\sec x + \tan x) + C \qquad \int \tan x dx = \left\{ \begin{array}{l} \text{Ln}(\sec x) + C \\ -\text{Ln}(\cos x) + C \end{array} \right\}$$

$$\int \cos x dx = \sin x + C \qquad \int \csc x dx = \text{Ln}(\csc x - \cot x) + C \qquad \int \cot x dx = \left\{ \begin{array}{l} -\text{Ln}(\csc x) + C \\ \text{Ln}(\sin x) + C \end{array} \right\}$$

$$\int \sec x \tan x dx = \sec x + C \qquad \int \sec^2 x dx = \tan x + C$$

$$\int \cos^2 x dx = \frac{1}{2} \sin x \cos x + \frac{1}{2} x + C \qquad \int \sin^2 x dx = -\frac{1}{2} \sin x \cos x + \frac{1}{2} x + C$$

Logs and Exponentials

$$\int e^x dx = e^x + C \qquad \int a^x dx = \frac{1}{\text{Ln} a} a^x + C$$

You do not need to know $\int \text{Ln}x dx$ but you should be able to get this integral by

Integration by Parts.

$$\int \text{Ln}x dx = x \text{Ln}x - x + C$$

$$\int \frac{dx}{x+b} = \text{Ln}(x+b) + C \qquad \int \frac{dx}{ax+b} = \frac{1}{a} \text{Ln}(ax+b) + C$$

$$\int \frac{dx}{x^2+a^2} = \frac{1}{a} \tan^{-1}\left(\frac{x}{a}\right) + C \qquad \int \frac{x dx}{x^2+a^2} = \frac{1}{2} \text{Ln}(x^2+a^2) + C$$

You should also know the effect of an ax in any of the preceding integral formulas.

$$\text{For example: } \int \cos(ax) dx = \frac{1}{a} \sin(ax) + C \qquad \int e^{ax} dx = \frac{1}{a} e^{ax} + C$$