

Factoring trinomials where  $a \neq 1$  using the “smiley face” method.

$$ax^2 + bx + c$$

- 1) Look for the GCF. If there is one, factor it out.
- 2) Set up the problem.
  - Write two parentheses
  - Put in signs. (First one comes down and the second one tells you same or different)
  - Put in your variables in either the front or the back of each parenthesis.
  - Draw inner and outer “smiley faces”.
- 3) Do the thought process:
  - Get the  $a \cdot c$
  - Think of two factors of  $a \cdot c$  that add or subtract (depending on the sign of  $c$ ) to get  $b$ .
  - The set of numbers that fits both stipulations is the “magic set”
- 4) Put the bigger number of the “magic set” on the inner smiley face. Put the smaller on the outer smiley face.
- 5) List the factors of  $a$  and then list out the factors of  $c$ . The factors of  $a$  can only go in the first spots of your parenthesis. The factors of  $c$  can only go in the last spots in the parenthesis.
- 6) Choose either  $a$  or  $c$  to place in first. It doesn't matter which you choose, but I suggest you choose the one that has the least set of factors. If they both have the same number of sets of factors, pick your favorite one. Whichever you choose, underline that spot in your parenthesis.
- 7) Look at the sets of factors of the one you choose. One of the factors has to go into the inner product and the other in the outer. The one that goes into the inner product, put it on the underline that is on the inner product. The one that goes into the outer product goes on the underline that is on the outer product.
- 8) Now fill in the remaining spots. The inner numbers need to multiply together to get the inner product. The outer numbers need to multiply together to get the outer product.
- 9) State the answer.
- 10) Check your answer by FOIL!

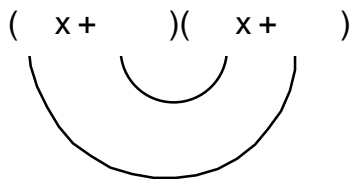
Example:

Factor  $3x^2 + 11x + 10$

1) **Look for the GCF. If there is one, factor it out.** No GCF

2) **Set up the problem.**

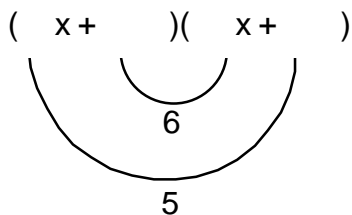
- Write two parentheses (      )(      )
- Put in signs. (First one comes down and the second one tells you same or different) ( +      )( +      )
- Put in your variables in either the front or the back of each parenthesis.  
( x+      )( x+      )
- Draw inner and outer “smiley faces”.



3) **Do the thought process:**

- Get the  $a \cdot c = 3 \cdot 10 = 30$
- Think of two factors of  $a \cdot c$  that add or subtract (depending on the sign of  $c$ ) to get  $b$ . \* 30 that + 11 [Multiply to get 30 that adds to get 11]
- The set of numbers that fits both stipulations is the “magic set”  $6 \cdot 5 = 30$  and  $6 + 5 = 11$ , so 6 and 5 are the “magic set”

4) **Put the bigger number of the “magic set” on the inner smiley face. Put the smaller on the outer smiley face.**



5) **List the factors of  $a$  and then list out the factors of  $c$ . The factors of  $a$  can only go in the first spots of your parenthesis. The factors of  $c$  can only go in the last spots in the parenthesis.**

Factors of  $a = 3 \cdot 1$

Factors of  $c = 10 \cdot 1$  and  $5 \cdot 2$

- 6) **Choose either a or c to place in first. It doesn't matter which you choose, but I suggest you choose the one that has the least set of factors. If they both have the same number of sets of factors, pick your favorite one. Whichever you choose, underline that spot in your parenthesis.**

I will put in the factors of a because there is only one set-  $3 \cdot 1$ . Since they make up the first term, I will underline the first spots in each parenthesis.

$$\left( \underline{\quad}x + \quad \right) \left( \underline{\quad}x + \quad \right)$$

6

5

- 7) **Look at the sets of factors of the one you choose. One of the factors has to go into the inner product and the other in the outer. The one that goes into the inner product, put it on the underline that is on the inner product. The one that goes into the outer product goes on the underline that is on the outer product.**

1 goes into both my inner and outer products. However, 3 only goes into 6. So I am going to put the 3 on the underline that is on the inner product.

$$\left( \underline{\quad}x + \quad \right) \left( \underline{3}x + \quad \right)$$

6

5

The 1 goes on the underline of the outer product.

$$\left( \underline{1}x + \quad \right) \left( \underline{3}x + \quad \right)$$

6

5

- 8) **Now fill in the remaining spots. The inner numbers need to multiply together to get the inner product. The outer numbers need to multiply together to get the outer product.**

The outer product is 5. I already have a 1. Therefore, I have to think "what times 1 gives me 5?" The answer is 5. So I put a 5 in the last spot of the second parenthesis.

$$\begin{array}{c} (\underline{1}x + \quad)(\underline{3}x + 5) \\ \text{6} \\ \text{5} \end{array}$$

The inner product is 6. I already have a 3. Therefore, I have to think, "what times 3 gives me 6?" The answer is 2. So I put a 2 in the last spot of the first parenthesis.

$$\begin{array}{c} (\underline{1}x + 2)(\underline{3}x + 5) \\ \text{6} \\ \text{5} \end{array}$$

9) **State the answer.**

$$\boxed{(x + 2)(3x + 5)}$$

(since  $1x = x$ , just write  $x$ )

10) **Check your answer by FOIL!**

$$\begin{array}{l} (x + 2)(3x + 5) \\ 3x^2 + 5x + 6x + 10 \\ 3x^2 + 11x + 10 \quad \checkmark \end{array}$$

Example:

$$\text{Factor } 28x^2 - 4xy - 5y^2$$

1) **Look for the GCF. If there is one, factor it out.** No GCF

2) **Set up the problem.**

- Write two parentheses (      )(      )

- Put in signs. (First one comes down and the second one tells you same or different) ( -      )( -      )

- Put in your variables in either the front or the back of each parenthesis.

Since  $x^2$  is the first term,  $x$  will go in the front spaces of both parenthesis. Since  $y^2$  is the last term,  $y$  will go in the last spaces of both parenthesis so it looks like:

$$( \quad x - \quad y ) ( \quad x - \quad y )$$

- Draw inner and outer “smiley faces”.

$$\begin{array}{c} ( \quad x - \quad y ) ( \quad x + \quad y ) \\ \text{ } \end{array}$$

3) **Do the thought process:**

- Get the  $a \cdot c = 28 \cdot 5 = 140$

- Think of two factors of  $a \cdot c$  that add or subtract (depending on the sign of  $c$ ) to get  $b$ . \* 140 that  $- 4$  [Multiply to get 140 that subtracts to get 4]

- The set of numbers that fits both stipulations is the “magic set”  $14 \cdot 10 = 140$  and  $14 - 10 = 4$ , so 14 and 10 are the “magic set”

4) **Put the bigger number of the “magic set” on the inner smiley face. Put the smaller on the outer smiley face.**

$$\begin{array}{c} ( \quad x - \quad y ) ( \quad x + \quad y ) \\ \text{ } \\ \text{ } \end{array}$$

5) **List the factors of  $a$  and then list out the factors of  $c$ . The factors of  $a$  can only go in the first spots of your parenthesis. The factors of  $c$  can only go in the last spots in the parenthesis.**

Factors of  $a = 28 \cdot 1, 14 \cdot 2, 7 \cdot 4$

Factors of  $c = 5 \cdot 1$



The inner product is 14. I already have a 1. Therefore, I have to think, "what times 1 gives me 14?" The answer is 14. So I put a 14 in the first spot of the second parenthesis.

$$(2x - \underline{1}y)(14x + \underline{5}y)$$

9) **State the answer.**

$$\boxed{(2x - y)(14x + 5y)}$$

(since  $1y = y$ , just write  $y$ )

10) **Check your answer by FOIL!**

$$\begin{aligned} &(2x - y)(14x + 5y) \\ &28x^2 + 10xy - 14xy - 5y^2 \\ &28x^2 - 4xy - 5y^2 \quad \checkmark \end{aligned}$$