Chapter 3: Solving Equations

Steps to solve an equation:

1st: Simplify each side of the equation

2nd: Decide which side you want the variable to be on

3rd: Using inverse operations, move the variable to that side.

4th: Once the variable is on one side, decide which operation would be done last in the order of operations if you were trying to evaluate that expression. Undo it by using it's inverse operation with the number.

5th: Continue working backwards from the order of operations using inverse operations until the variable is alone. ("Alone" means only positive one multiplying/dividing the variable and nothing adding/subtracting it.)

Rules to solving equations:

- Both sides must be completely simplified before you can start solving.
- Always work down. Never work across.
- Decimals can only be used if there are decimals in the equation or you are working with money.
- You must work with improper fractions only, never with mixed numbers!
- Solutions must be written in { }'s and boxed/highlighted or circled.
- Fractional solutions must be reduced or changed into mixed numbers.
- Anytime there is a fraction, your first step after simplifying must be getting rid of the denominator.
- Multiplying by negative one or dividing by negative one will change negative x into positive x.

Rules to check equation:

- In the ORIGINAL equation, replace the variable with the solution. (If the solution is a mixed number, use the improper fraction because we never work with mixed numbers.)
- Using the order of operations and working down, simplify the left hand side (LHS).
- Using the order of operations and working down, simplify the right hand side (RHS).
- Once both sides have been simplified down to a number, the type of statement determines whether the answer you got is the solution:
    - A true statement (LHS = RHS) means your answer is the solution so box it and go on.
    - A false statement (LHS $\neq$ RHS) means you have made an error and you have to go back and redo.
- Remember you are not using the order of operation if you:
    - Use equation solving techniques while checking.
    - Use the distributive property while checking. [You only distribute when you can't simplify what is in the parentheses. Since you will only have numbers, you should be able to simplify the parentheses.]

3 – 1 Solving Equations with Addition and Subtraction

Know the Addition Property of Equality, Subtraction Property of Equality, and the definition of equivalent equations.

Refer to the Chapter 3 solving equations reference sheet before attempting to solve or check any equation.

Addition and Subtraction are inverse operations. That means they will "undo" each other. If you are trying to "undo" addition, use subtraction. If you are trying to "undo" subtraction, use addition.

Solve and check.

1) $x - 4 = 10$ What is happening to x? **4 is being subtracted.**
What is the inverse operation of subtract? **Add**

$x - 4 = 10$ Therefore you need to add 4 to both sides.
$\quad + 4 \quad + 4$

$x \quad = 14$ The – 4 and + 4 result in zero, which leaves us with x on the LHS.
The + 10 and + 4 result in 14.

The final answer is $\boxed{\{14\}}$.

Check x = 14:

$(\quad) - 4 = 10$ Rewrite the ORIGINAL equation using open parentheses for x.
$(14) - 4 = 10$ Replace the open parentheses with the solution.
Simplify the left side using the order of operations.
$14 + (-4) = 10$ Subtraction Rule
$10 = 10$ Since the LHS = RHS, 14 is the solution!

2) $3 = 9 + x$ What is happening to x? **9 is being added.**
What is the inverse operation of addition? **Subtraction**

$3 = 9 + x$ Therefore, you need to subtract 9 from both sides.
$-9 \quad -9$

$-6 = x$ The + 9 and – 9 result in zero, which leaves us with x on the RHS.
The + 3 and – 9 result in –6 (Signs are different- subtract and take larger sign.)

The final answer is $\boxed{\{-6\}}$.

Check x = -6:

$3 = 9 + (\quad)$ Rewrite the ORIGINAL equation using open parentheses for x.
$3 = 9 + (-6)$ Replace the open parentheses with the solution and simplify the right side using the order of operations.
$3 = 3$ Different signs – subtract and take larger sign.
Since the LHS = RHS, 14 is the solution!

3) $18 - x = 11$ — Since x is following a minus sign, we need to move it to the other side by adding x to both sides. Do NOT put it under 18 because they are not like terms. Only put like terms under each other.

$$\begin{array}{rcl} 18 - x &=& 11 \\ +x & & +x \\ 18 &=& 11 + x \end{array}$$

The – x and + x result in zero, which leaves us 18 on the LHS. Since 11 and x **can't** be combined, we have 11 + x on the RHS. Now we can start solving for x just like example 2.

$$\begin{array}{rcl} 18 &=& 11 + x \\ -11 & & -11 \\ 7 &=& x \end{array}$$

Subtract 11 from both sides to get x by itself.

The + 11 and – 11 result in zero, leaving us with x on the RHS. The + 18 and – 11 result in 7 (Signs are different- subtract and take larger sign.)

The final answer is {7}.

Check x = 7:

18 – ( ) = 11	Rewrite the **ORIGINAL** equation using open parentheses for x.
18 – (7) = 11	Replace the open parentheses with the solution and simplify the right side using the order of operations.
18 + (-7) = 11	Use subtraction Rule
11 = 11	Different signs means subtract and take the larger sign. Since the LHS = RHS, 7 is the solution!

3 – 2 Solving Equations with Multiplication and Division

Know the Multiplication Property of Equality and the Division Property of Equality.

Multiplication and division are inverse operations. That means they will "undo" each other. If you are trying to "undo" multiplication, use division. If you are trying to "undo" division, use multiplication.

Solve and check.

1) $24 = -3x$ — What is happening to x? **Multiplied by –3**
What is the inverse operation of multiply? **Divide**

$$\frac{24}{-3} = \frac{-3x}{-3}$$

Divide each side by –3 because –3 was the number multiplying x.

$-8 = x$ — 24 ÷ -3 = -8 (different signs- quotient negative), -3 ÷ -3 = 1 and 1 • x = x. So we are left with x on the RHS.

The final answer is {-8}.

Check x = -8:

24 = -3( )	Rewrite the ORIGINAL equation using open parentheses for x.
24 = -3(-8)	Replace the open parentheses with the solution and simplify the RHS using the order of operations. Same signs- product is positive.
24 = 24	Since the LHS = RHS, -8 is the solution!

2) $-\frac{x}{5} = -3$

Notice there is a negative on the LHS. That means –5 is dividing x.

To get rid of –5 dividing x, you have to multiply both sides by –5

$\left(-\frac{x}{5}\right) = (-3)$

When you multiply both sides, put each side in parentheses.

$\frac{-5}{1} \bullet \left(-\frac{x}{5}\right) = (-3) \bullet -5$

Now, multiply each by –5. Multiply it as a fraction on the LHS.

x = 15

LHS: Same signs- product is positive. The 5's cancel because of Mult. Inv. Prop. The result is x.
RHS: Same sign- product positive. The result is 15.

The final answer is $\boxed{\{15\}}$.

Check x = 15:

$-\frac{(\ \ )}{5} = -3$

Rewrite the ORIGINAL equation using open parentheses for x.

$-\frac{(\ 15\ )}{5} = -3$

Replace the open parentheses with the solution and simplify the

LHS using the order of operations. Diff signs- quotient is negative.

-3 = -3

Since the LHS = RHS, 15 is the solution!

3) 6¾ = 5x

NEVER work w/mixed numbers! Change to an improper fraction.

$\frac{27}{4} = 5x$

4 • 6 + 3 = 27. 27 is the numerator and 4 is the denominator.

What is happening to x? **5 is multiplying x.**
What is the inverse operation of multiplying? **Dividing**

$\left(\frac{27}{4}\right) = \left(\frac{5x}{1}\right)$

Rewrite equation with each side is in parentheses & both are fractions

$\frac{1}{5} \bullet \left(\frac{27}{4}\right) = \left(\frac{5x}{1}\right) \bullet \frac{1}{5}$

Remember, dividing is multiplying the reciprocal. So instead of dividing by 3, we are going to multiply by $\frac{1}{3}$.

$\frac{27}{20} = x$

RHS: Because of Mult. Inv. Prop. this reduces to 1 • x = x
LHS: Nothing cancels so multiply straight across.

$1\frac{7}{20} = x$

The final answer can't be an improper fraction, convert to mixed #.

The final answer is $\boxed{\{1\frac{7}{20}\}}$. (The check on the next page)

Check x = $1\frac{7}{20}$:

Never check with mixed numbers only with improper fractions or integers. So convert this back to an improper fraction.

Use x = $\frac{27}{20}$

$6\frac{3}{4} = 5(\quad)$ Rewrite the ORIGINAL equation using open parentheses for x.

$6\frac{3}{4} = 5(\frac{27}{20})$ Replace the open parentheses with the solution as an improper fraction.

$6\frac{3}{4} = \frac{5}{1}\left(\frac{27}{20}\right)$ Rewrite so that 5 is over one to make it easier to simplify the RHS.

LHS: Rewrite 6¾ as an improper fraction $\frac{27}{4}$

RHS: Cancel and multiply straight across.

$\frac{27}{4} = \frac{27}{4}$ Since the LHS = RHS, $1\frac{7}{20}$ is the solution!

3-3 Solving Multi-Step Equations

To solve multi-step equations use inverse operations going backwards from the order of operations.

Solve and check.

1) $\frac{x+7}{4} = 5$ If you knew the value of x, how would you simplify the LHS?

Add 7 and then divide by 4.

To solve, go backwards. So you have to take care of divide by 4 1st. The inverse operation of divide is multiply. So, multiply each side by 4.

$\left(\frac{x+7}{4}\right) = (5)$ Remember whenever you multiply both sides, use parentheses.

$\frac{4}{1}\bullet\left(\frac{x+7}{4}\right) = (5)\bullet 4$ On the LHS, write 4 as a fraction. It isn't necessary on the RHS.

$\frac{\not{4}}{1}\bullet\left(\frac{x+7}{\not{4}}\right) = 20$ LHS: The fours cancel. RHS: 5 • 4 = 20

$x + 7 = 20$ Now, subtract 7 from both sides.

$-7 \quad -7$

$x = 13$

The final answer is $\boxed{\{13\}}$. (The check on the next page)

Check x = 13.

$\frac{(\quad) + 7}{4} = 5$	Rewrite the ORIGINAL equation using open parentheses for x.
$\frac{(13) + 7}{4} = 5$	Replace the open parentheses with the solution
	Simplify the RHS using the order of operations.
$\frac{20}{4} = 5$	As we side before, that would be Add 7
$5 = 5$	and then Divide by 4
	Since the LHS = RHS, 15 is the solution!

2) $21 - 5x = 31$

If you knew the value of x, how would you simplify the LHS?

Multiply by –5 then add 21.

To solve, go backwards. So you have to take care of add 21 1st. The inverse operation of add is subtract. So, subtract each side by 21. Remember the sign of 21 is +. Do NOT look behind a term to determine it's sign.

$$\begin{aligned} 21 - 5x &= 31 \\ -21 \qquad &\;\; -21 \\ \frac{-5x}{-5} &= \frac{10}{-5} \\ x &= -2 \end{aligned}$$

Now divide both sides by –5.

The final answer is $\boxed{\{-2\}}$.

Check x = -2.

$21 - 5(\quad) = 31$	Rewrite the ORIGINAL equation using open parentheses for x.
$21 - 5(-2) = 31$	Replace the open parentheses with the solution and simplify the RHS using the order of operations.
$21 - (-10) = 31$	Be very careful about the signs!
$21 + 10 = 31$	Subtraction Rule
$31 = 31$	Since the LHS = RHS, -2 is the solution!

3 – 5 Solving Equations with variables on both sides

To solve an equation, you are trying to find the value of x. Therefore, you need to get all your x's on one side. Remember to move a term from one side to the other, you will either use subtraction or addition.

Solve and check.

Ex. 1:

$2x + 15 = 3x$	On the LHS, there is an x term & a number. On the RHS, the 3x is the only term. Since you want all the x terms together, move the 2x
$-2x \quad\quad -2x$	onto the RHS. To move the 2x, we will subtract it from both sides.
$15 = x$	The x terms on the left cancel and 15 is left. The RHS becomes x because 3x – 2x = x
$\boxed{\{15\}}$	Therefore, the final solution is 15.

Ex. 2:

$3(2x - 1) + 6 = 4x - 5$	Before solving, you must simplify each side of the equation.
$6x - 3 + 6 = 4x - 5$	Use the distributive property to get rid of the parentheses.
$6x + 3 = 4x - 5$	Combing like terms (– 3 + 6 = 3) And now that both sides are simplified, we are ready to start solving! Because there are x terms on both sides, we want to get them together. We can put them on either side. Let's get them on the LHS.
$6x + 3 = 4x - 5$	To get the x terms on the LHS, subtract 4x from both sides.
$-4x \quad\quad -4x$	LHS: 6x – 4x = 2x and the + 3 comes down.
$2x + 3 = -5$	RHS: 4x – 4x = zero, and the – 5 comes down.
$-3 \quad -3$	Subtr.3 from both sides to get LHS to be 2x. RHS:-5 and -3 = -8
$\frac{2x}{2} = \frac{-8}{2}$	Now Divide both sides by 2.
$x = -4$	The final solution is -4.
$\boxed{\{ -4 \}}$	

To check plug into the ORIGINAL EQUATION and use the ORDER OF OPERATIONS to simplify each side!!!

Check x = -4

$3(2(\quad) - 1) + 6 = 4(\quad) - 5$

$3(2(-4) - 1) + 6 = 4(-4) - 5$

$3(-8 - 1) + 6 = -16 - 5$ (One simplification per side)

$3(-9) + 6 = -21$ Do NOT use the distributive property! Simplify inside the parentheses. Remember -16 – 5 = -16 + -5

$-27 + 6 = -21$

$-21 = -21$

Sometimes when you are solving an equation, the variables cancel.

Ex. 3:

$3x + 7 = 3x$	If you subtract 3x from both sides, look what happens.
$-3x \qquad -3x$	
$7 = 0$	The x's cancel (because of the Additive Inverse Property) and you no longer have a variable in your equation. Therefore, you can't say x = a specific number.

Look at the result, it is $7 = 0$. Is this true? NO, $7 \neq 0$ Since the result is false, the final answer is NO SOLUTION. That means there is no number that can replace the variable to make this statement true.

Ex. 4:

$8x + 11 = 2(4x - 7) + 25$	Simplify the RHS by distributive property
$8x + 11 = 8x - 14 + 25$	Combine like terms
$8x + 11 = 8x + 11$	Get x terms on one side
$-8x \qquad -8x$	
$11 = 11$	x terms cancel leaving you with a TRUE statement.

The final answer is IDENTITY.