12-2 Multiplying Rational Expression

To multiply rational expressions:

1) Factor each numerator and each denominator
2) Cancel any common factors
3) Multiply straight across (leave in factored form)

Simplify. Assume no denominator equals zero.

Example 1: $\frac{10x^2-360}{8x+20} \bullet \frac{2x^2+17x+30}{18-3x}$

1) Factor each numerator and each denominator

1st fraction's numerator:

Factor out the GCF of 10. $10(x^2 - 36)$

A difference of squares is left $10(x + 6)(x - 6)$

1st fraction's denominator:

Factor out the GCF of 4 $4(2x + 5)$

2nd fraction's numerator:

There is no GCF.

This a trinomial where a ≠ 1 $(x + 6)(2x + 5)$

2nd fraction's denominator:

Factor out the GCF of 3 $3(6 - x)$

$$\frac{10(x+6)(x-6)}{4(2x+5)} \bullet \frac{(x+6)(2x+5)}{3(6-x)}$$

2) Cancel any common factors

Starting with the constant terms. The 10 in the numerator and 4 in the denominator share the common factor of 2. So they reduce to $\frac{5}{2}$.

$$\frac{\cancel{10}^{5}(x + 6)(x - 6)}{\cancel{4}_{2}(2x + 5)} \bullet \frac{(x + 6)(2x + 5)}{3(6 - x)}$$

There is a (2x + 5) in both the top and the bottom so they will cancel to equal 1.

$$\frac{\cancel{10}^{5}(x+6)(x-6)}{\cancel{-4}^{2}(\cancel{2x+5})} \bullet \frac{(x+6)(\cancel{-2x+5})}{3(6-x)}$$

There is an (x – 6) on the top and an (6 – x) on the bottom. Therefore, they cancel to equal –1.

$$\frac{\cancel{10}^{5}(x+6)(\cancel{x-6})^{-1}}{\cancel{-4}^{2}(\cancel{2x+5})} \bullet \frac{(x+6)(\cancel{-2x+5})}{3(\cancel{6-x})^{1}}$$

Since the (x + 6)'s are both on the top, they cannot cancel. One must be on top and the other on the bottom to cancel.

3) Multiply straight across (leave in factored form)

$$\frac{\cancel{10}^{5}(x+6)(\cancel{x-6})^{-1}}{\cancel{-4}^{2}(\cancel{2x+5})} \bullet \frac{(x+6)(\cancel{-2x+5})}{3(\cancel{6-x})^{1}} = \frac{5 \bullet (x+6) \bullet -1}{2} \bullet \frac{(x+6) \bullet 1}{3}$$

Remember $(x + 6)(x + 6) = (x + 6)^2$

$$\boxed{\frac{-5(x+6)^2}{6}}$$

Example 2: $\dfrac{10x^2y^3}{x^2+8x+7} \bullet \dfrac{3x^2+21x}{8xy^5}$

1) Factor each numerator and each denominator

1st fraction's numerator:
It is a monomial. So no factoring to do.

1st fraction's denominator:
There is no GCF.
It is a trinomial where a = 1. $(x + 7)(x + 1)$

2nd fraction's numerator:
Factor out the GCF of 3x $3x(x + 7)$

2nd fraction's denominator:
It is a monomial. So no factoring to do.

$$\frac{10x^2y^3}{(x+7)(x+1)} \bullet \frac{3x(x+7)}{8xy^5}$$

2) Cancel any common factors

There is a (x +7) in both the top and the bottom so they will cancel to equal 1.

$$\frac{10x^2y^3}{(\cancel{x+7})(x+1)} \bullet \frac{3x(\cancel{x+7})}{8xy^5}$$

There is a common factor of 2 in 10 and 8.

$$\frac{5\ \cancel{10}\,x^2y^3}{(\cancel{x+7})(x+1)} \bullet \frac{3x(\cancel{x+7})}{\cancel{8}\,xy^5\ 4}$$

3) Multiply straight across (leave in factored form)

Let's combine what we have and then take care of the monomials.

$$\frac{\overset{5}{\cancel{10}}x^2y^3}{(\cancel{x+7})(x+1)} \bullet \frac{3x(\cancel{x+7})}{\underset{4}{\cancel{8}}xy^5} = \frac{5 \bullet x^2y^3 \bullet 3x \bullet 1}{(x+1) \bullet 4 \bullet xy^5} = \frac{15 \bullet x^2y^3 \bullet x}{4xy^5(x+1)}$$

Remember, when mulitplying with like bases, ADD the EXPONENTS.

$$\frac{15x^{2+1}y^3}{4xy^5(x+1)} = \frac{15x^3y^3}{4xy^5(x+1)}$$

Remember, when dividing with like bases, SUBTRACT the EXPONENTS.

$$\frac{15x^{3-1}}{4y^{5-3}(x+1)}$$

So the final answers is: $\boxed{\frac{15x^2}{4y^2(x+1)}}$