12-1 Simplifying Rational Expressions

A **rational expression** is an algebraic fraction whose numerator and denominator are polynomials.

Since a fraction cannot have a denominator of zero, any number that will make the denominator zero must be excluded. To find the excluded values of a rational expression, set the denominator equal to zero, factor and solve. (Like 10-6).

Recall 10-6:

(1) Get all the terms on one side so the other side is zero.
(2) Factor the polynomial completely.
(3) Set each multiplier equal to zero and solve for the variable.
(4) List solutions least to greatest in { }'s
(5) Check each solution in the original equation.

Find the excluded values.

Example 1: $\frac{3}{10x^2y}$

To find the excluded values, set the denominator equal to zero and solve like 10-6
$10x^2y = 0$

Since no factoring needs to be done, we can set each multiplier equal to zero and solve.
$10 = 0$ $x^2 = 0$ $y = 0$

$10 = 0$ gets thrown out. There is no variable to solve for.

If $x^2 = 0$, then $x = 0$.

No solving needed on $y = 0$.

Therefore the excluded values are that x and y cannot be 0, so the answer is:
$x \neq 0$ and $y \neq 0$

Example 2: $\dfrac{x+3}{x^2+4x-12}$

To find the excluded values, set the denominator equal to zero and solve like 10-6

$x^2 + 4x - 12 = 0$

FACTOR:

There is no GCF. This is a trinomial where a = 1.

$(x + 6)(x - 2) = 0$ Thought Process: Multiply to get 12 and subtracts to get 4.

We can set each multiplier equal to zero and solve.

$x + 6 = 0$	and	$x - 2 = 0$
$-6 \quad -6$		$+2 \quad +2$
$x = -6$		$x = 2$

Therefore the excluded values are –6 and 2 and so the answer is:

$\boxed{x \neq -6, 2}$

***

Remember that a factor is a multiplier.

2 and 6 are factors of 12 because they multiply to give you 12. Therefore, 2 and 6 are each multipliers.

(x + 6) and (x – 2) are factors of $x^2 + 4x - 12$. Therefore, (x + 6) and (x – 2) are each multipliers.

3, (x + 5) and (x – 5) are factors of $3x^2 - 75$. Because if $3x^2 - 75$ was factored completely it would equal 3(x + 5)(x – 5).

How many multipliers are there and what are they?

Example 3: (3x + 11)(x + 9)

There are 2 and they are (3x + 11) and (x + 9)

Example 4: $x(x - 1)(5x + 6)(2x^2 + x + 13)$

There are 4 and they are x, (x – 1), (5x + 6) and $(2x^2 + x + 13)$

***

To simplify a rational expression you must eliminate any factor that both the numerator and denominator have in common. <u>Therefore, factor the numerator and denominator completely before canceling</u>. You may only cancel factors/multipliers. Remember a sum/difference is one unit. You can only cancel the whole sum/difference. Suggestion- always put parenthesis around a sum/difference while working.

Simplify (leave in factored form)

Example 5: $\frac{2x+14}{x^2+2x-35}$

First, factor the numerator and the denominator completely.

The numerator has a GCF of 2, factor that out and you are left with 2(x + 7). Now it is factored completely.

The denominator has no GCF. It is a trinomial where a = 1. Thought process: Multiplies to get 35 and subtracts to get 2.

$$\frac{2x+14}{x^2+2x-35}=\frac{2(x+7)}{(x+7)(x-5)}$$

Determine the factors/multipliers in the numerator:

2 mulipliers- 2 and (x + 7)

Determine the factors/multipliers in the denominator:

2 mulipliers- (x + 7) and (x – 5)

Cancel.

Since (x + 7) appears in both the numerator and denominator, it cancels to equal 1.

$$\frac{2\cancel{(x+7)}}{\cancel{(x+7)}(x-5)}$$

Answer:

$$\boxed{\frac{2}{x-5}}$$

a) State the excluded values and b) Simplify (leave in factored form).

Example 6: $\dfrac{4x^2-13x+3}{16x^2-1}$

a) To find the excluded values, set the denominator equal to zero and solve like 10-6

$16x^2 - 1 = 0$

FACTOR:

There is no GCF. This is a binomial so we check to see if it is a difference of squares. It is because it is a difference, $16x^2 = (4x)^2$ and $1 = (1)^2$.

$(4x + 1)(4x - 1) = 0$

We can set each multiplier equal to zero and solve.

$4x + 1 = 0$	and	$4x - 1 = 0$
$-1 \quad -1$		$+1 \quad +1$
$\dfrac{4x}{4} = \dfrac{-1}{4}$		$\dfrac{4x}{4} = \dfrac{1}{4}$
$x = -\frac{1}{4}$		$x = \frac{1}{4}$

Therefore the excluded values are $-\frac{1}{4}$ and $\frac{1}{4}$ and so the answer is:

$\boxed{x \neq -\frac{1}{4}, \frac{1}{4}}$

Once the excluded values have been stated, simplify. It is important that you do the excluded values first.

b) First, factor the numerator and the denominator completely.

The numerator does not have a GCF. It is a trinomial where $a \neq 1$. Thought process: $a \bullet c = 12$, so think multiplies to get 12 and adds to get 13.

The denominator has already been factored from part a.

$$\frac{4x^2-13x+3}{16x^2-1} = \frac{(x-3)(4x-1)}{(4x+1)(4x-1)}$$

Determine the factors/multipliers in the numerator:

2 mulipliers- (x – 3) and (4x – 1)

Determine the factors/multipliers in the denominator:

2 mulipliers- (4x + 1) and (4x – 1)

Cancel.

Since (4x – 1) appears in both the numerator and denominator, it cancels to equal 1.

$$\frac{(x-3)\cancel{(4x-1)}}{(4x+1)\cancel{(4x-1)}}$$

Therefore the answer is $\boxed{\frac{x-3}{4x+1}}$

a) x ≠ -¼, ¼

b) $\frac{x-3}{4x+1}$

Example 7: $\frac{20-6x-2x^2}{4x^2+32x-80}$

a) To find the excluded values, set the denominator equal to zero and solve like 10-6

$4x^2 + 32x - 80 = 0$

FACTOR:

There is a GCF of 4. Factor it out.

$4(x^2 + 8x - 20) = 0$

Now, there is a trinomial where a = 1. Thought process: multiplies to get 20 and subtracts to get 8.

4(x + 10)(x – 2) = 0

We can set each multiplier equal to zero and solve.

~~4 = 0~~	x + 10 = 0	and	x – 2 = 0
Thrown	– 10 – 10		+ 2 + 2
Out b/c	x = -10		x = 2
no variable			
to slove for.			

Therefore the excluded values are –10 and 2 and so the answer is:
$x \neq -10, 2$

b) First, factor the numerator and the denominator completely.

The numerator has a GCF of 2. Factor it out.

$2(10 - 3x - x^2)$

The trinomial left has the x^2 term is in the last spot. Therefore, factor as if a ≠ 1.
Thought process: a • c = 10, so think multiplies to get 10 and subtracts to get 3.

$2(2 - x)(5 + x)$

The denominator has already been factored from part a.

$$\frac{20-6x-2x^2}{4x^2+32x-80}=\frac{2(2-x)(5+x)}{4(x+10)(x-2)}$$

Determine the factors/multipliers in the numerator:

3 mulipliers- 2, (2 – x) and (5 + x)

Determine the factors/multipliers in the denominator:

3 mulipliers- 4, (x + 10) and (x – 2)

Cancel.

Remember (x – y) = -1(y – x)
So, if your differences are backwards from each other, they cancel and you are left with –1 (negative one).

Remember (x + y) = (y + x)
However, if your sums are backwards, they cancel to be 1 (positive one)

There are two things to cancel here. There is a 2 in the numerator and a 4 in the denomiator. They reduce to ½. As long as there is somehting else left in the numerator, we do not have to write the 1. Since (2 – x) appears in numerator and (x – 2) appears denominator, it cancels to equal -1.

$$\frac{\overset{1}{\cancel{2}}(\overset{-1}{\cancel{2 - x}})(5 + x)}{\underset{2}{\cancel{4}}(x + 10)(\cancel{x - 2})} = \frac{1 \bullet -1 \bullet (5 + x)}{2 \bullet (x + 10)}$$

Since there is no constant in the numerator, we will put the negative out in front of the fraction.

Therefore the answer is $\boxed{-\frac{5+x}{2(x+10)}}$

a) $x \neq -10, 2$

b) $-\frac{5+x}{2(x+10)}$