Work Problems

I. Find the work done by a force of $F(x)$ newtons along the $x-$axis from $x=a$ meters to $x=b$ meters.

1. $F(x)=xe^{-\frac{x}{3}}$ $\quad a=0$ $\quad b=5$

$W=\int_0^5 xe^{-\frac{x}{3}}dx=\boxed{4.467\,J}$

2. $F(x)=x\sin\left(\frac{p\,x}{4}\right)$ $\quad a=0$ $\quad b=3$

$W=\int_0^3 x\sin\left(\frac{p\,x}{4}\right)dx=\boxed{3.845\,J}$

3. $F(x)=x\sqrt{9-x^2}$ $\quad a=0$ $\quad b=3$

$W=\int_0^3 x\sqrt{9-x^2}\,dx=\boxed{9\,J}$

4. $F(x)=e^{\sin x}\cos x+2$ $\quad a=0$ $\quad b=10$

$W=\int_0^{10} e^{\sin x}\cos x dx=\boxed{19.580\,J}$

II. **Hooke's Law**

1. A spring has a natural length of 10 inches. An 800 pound force stretches the spring to 16 inches. How much work is done stretching the spring from 10 inches to 12 inches?

$F=kx \quad\rightarrow\quad 800=k(16) \quad\rightarrow\quad K=50$

$F(x)=50x$

$W=\int_{10}^{12}50xdx=\boxed{1100\,inch\cdot lbs}$

2. A bathroom scale is compressed $\frac{1}{16}in$ when a 150 pound persond stands on it. How much work is done in compressing the scale $\frac{1}{8}in$?

$150=k\left(\frac{1}{16}\right) \quad\rightarrow\quad k=2400$

$W=\int_0^{\frac{1}{8}}2400xdx=\boxed{18.75\,inch\cdot lbs}$

3. A force of 5 pounds compresses a 15 inch spring a total of 4 inches. How much work is dones in compressing the spring 7 inches?

$5 = k(4) \quad \rightarrow \quad k = 1.25$

$W = \int_0^7 1.25x dx = \boxed{30.625\, inch \cdot lbs}$

4. A force of 200 pounds stretches a spring 2 feet on a mechanical device for driving fence posts. Find the work done in stretching the spring the required 2 feet.

$200 = k(2) \quad \rightarrow \quad k = 100$

$W = \int_0^2 100x dx = \boxed{200\ ft \cdot lbs}$

5. A force of 600 newtons stretches a spring 1 meter. How much work is done in stretching the spring from 0.9 meters to 1.5 meters?

$600 = k(1) \quad \rightarrow \quad k = 600$

$W = \int_{0.9}^{1.5} 600x dx = \boxed{432\ J}$

III. **Newton's Law of Gravitation**

1. If a space module weighs 12 tons on the surface of the earth, how much work is done in propelling the module to a height of 500 miles?

$F = \frac{C}{x^2} \quad \rightarrow \quad 12 = \frac{C}{(4000)^2} \quad \rightarrow \quad C = 192,000,000$

$W = \int_{4000}^{4500} \frac{192,000,000}{x^2} dx = \boxed{5333.333\, mile \cdot tons}$

2. If a space module weighs 4 tons on the surface of the moon, how much work is done in propelling the module to a height of 50 miles? [The radius of the moon is 1100 miles]

$4 = \frac{C}{(1100)^2} \quad \rightarrow \quad C = 4,840,000$

$W = \int_{1100}^{1150} \frac{4840000}{x^2} dx = \boxed{191.304\, mile \cdot tons}$