Shell Method Problems

Let **R** be the region bounded by the given curves. Find the volume when **R** is revolved about the given axes.

1. $y = x - x^2, y = 0$

$a)\quad y - axis$

$R = x \quad h = x - x^2$

$$V = 2p\int_0^1 \left(x(x - x^2)\right)dx = 2p\int_0^1 \left(xy_1\right)dx = \boxed{\frac{p}{6} = .524}$$

$b)\quad x = -2$

$R = x + 2 \quad h = x - x^2$

$$V = 2p\int_0^1 \left((x+2)(x - x^2)\right)dx = 2p\int_0^1 \left((x+2)y_1\right)dx = \boxed{\frac{5p}{6} = 2.618}$$

$c)\quad x = 3$

$R = 3 - x \quad h = x - x^2$

$$V = 2p\int_0^1 \left((3-x)(x - x^2)\right)dx = 2p\int_0^1 \left((3-x)y_1\right)dx = \boxed{\frac{5p}{6} = 2.618}$$

2. $y = 4 - x^2, y = 2 - x$

$a)\quad x = -2$

$R = x + 2 \quad h = 4 - x^2 - (2 - x)$

$$V = 2p\int_{-1}^2 \left((x+2)(4 - x^2 - (2 - x))\right)dx = 2p\int_{-1}^2 \left((x+2)(y_1 - y_2)\right)dx = \boxed{\frac{45p}{2} = 70.686}$$

$b)\quad x = 4$

$R = 4 - x \quad h = 4 - x^2 - (2 - x)$

$$V = 2p\int_{-1}^2 \left((4-x)(4 - x^2 - (2 - x))\right)dx = 2p\int_{-1}^2 \left((4-x)(y_1 - y_2)\right)dx = \boxed{\frac{63p}{2} = 98.960}$$

3. $y = x^2 + 1, y = x + 3$

$a)\quad x = -1$

$R = x + 3 \quad h = x + 3 - (x^2 + 1)$

$$V = 2p\int_{-1}^{2}\left((x+1)(x+3-(x^2+1)\right)dx = 2p\int_{-1}^{2}\left((x+1)(y_2 - y_1)\right)dx = \boxed{\frac{27p}{2} = 42.412}$$

$b)\quad x = 5$

$R = 5 - x \quad h = x + 3 - (x^2 + 1)$

$$V = 2p\int_{-1}^{2}\left((5-x)(x+3-(x^2+1))\right)dx = 2p\int_{-1}^{2}\left((5-x)(y_2 - y_1)\right)dx = \boxed{\frac{81p}{2} = 127.235}$$

4. $y = 6 - x^2, y = x + 3$

$A = -2.302775638 \quad B = 1.302775638$

$a)\quad x = -3$

$R = x + 3 \quad h = 6 - x^2 - (x + 3)$

$$V = 2p\int_{-2}^{1}\left((x+3)(6-x^2-(x+3))\right)dx = 2p\int_{-2}^{1}\left((x+3)(y_1 - y_2)\right)dx = \boxed{\frac{75p}{2} = 117.810}$$

$b)\quad x = 3$

$R = 3 - x \quad h = 6 - x^2 - (x + 3)$

$$V = 2p\int_{-2}^{1}\left((3-x)(6-x^2-(x+3))\right)dx = 2p\int_{-2}^{1}\left((3-x)(y_1 - y_2)\right)dx = \boxed{\frac{105p}{2} = 164.934}$$

5. $y = 5\cos(x), y = x + 2, y - axis$

$A = 0 \quad B = .9417736923$

$a)\quad y - axis$

$R = x \quad h = 5\cos(x) - (x + 2)$

$$V = 2p\int_{A}^{B}\left(x(5\cos(x)-(x+2)\right)dx = 2p\int_{A}^{B}\left(x(y_1 - y_2\right)dx = \boxed{3.669}$$

$b)\quad x = -1$

$R = x + 1 \quad h = 5\cos(x) - (x + 2)$

$$V = 2p\int_{A}^{B}\left((x+1)(5\cos(x)-(x+2)\right)dx = 2p\int_{A}^{B}\left((x+1)(y_1 - y_2\right)dx = \boxed{14.451}$$

$c)\quad x = 2$

$R = 2 - x \quad h = 5\cos(x) - (x + 2)$

$$V = 2p\int_{A}^{B}\left((2-x)(5\cos(x)-(x+2)\right)dx = 2p\int_{A}^{B}\left((2-x)(y_1 - y_2\right)dx = \boxed{17.894}$$

6. $y=\sqrt{x}, y=2$ and the $y-$axis.

$a)\quad y-axis$

$R=x \quad h=2-\sqrt{x}$

$$V=2p\int_0^4\left(x(2-\sqrt{x}\right)dx=2p\int_0^4\left(x(2-y_1\right)dx=\boxed{20.106}$$

$b)\quad x=-2$

$R=x+2 \quad h=2-\sqrt{x}$

$$V=2p\int_0^4\left((x+2)(2-\sqrt{x}\right)dx=2p\int_0^4\left((x+2)(2-y_1\right)dx=\boxed{53.617}$$

$c)\quad x=5$

$R=5-x \quad h=2-\sqrt{x}$

$$V=2p\int_0^4\left((5-x)(2-\sqrt{x})\right)dx=2p\int_0^4\left((5-x)(2-y_1\right)dx=\boxed{63.670}$$

7. $y=e^x, y=x, x=1$ and $x=2$

$a)\quad y-axis$

$R=x \quad h=e^x-x$

$$V=2p\int_1^2\left(x(e^x-x)\right)dx=2p\int_1^2\left(x(y_1-y_2\right)dx=\boxed{31.766}$$

$b)\quad x=-2$

$R=x+2 \quad h=e^x-x$

$$V=2p\int_1^2\left((x+2)(e^x-x)\right)dx=2p\int_1^2\left(x+2)(y_1-y_2\right)dx=\boxed{71.611}$$

$c)\quad x=4$

$R=4-x \quad h=e^x-x$

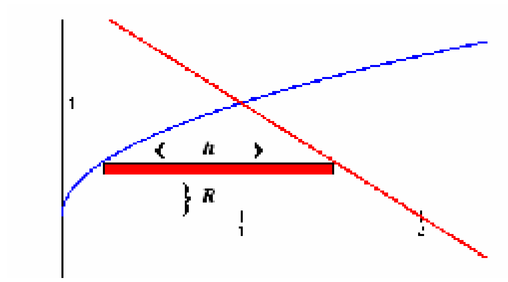
$$V=2p\int_1^2\left((4-x)(e^x-x)\right)dx=2p\int_1^2\left(4-x)(y_1-y_2\right)dx=\boxed{47.924}$$

Special Cases

#1: Let **R** be the region bounded by

$y=\sqrt{x},\ y=2-x$ and the $x-$axis.

Find thevolume of the solids generated when R is revolved about



a) the $x-axis$

Here the box is parallel so we use the Shell Method.

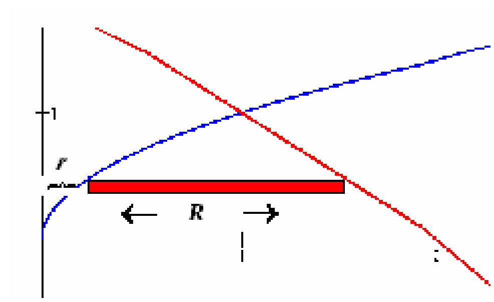
Solving the about equations for x we get

$x=y^2 \quad x=2-y.$ The intersecting points are at $x=1$ and $y=1.$

$R=y \quad h=2-y-y^2$

$$V=2p\int_0^1 y(2-y-y^2)dy=\boxed{\frac{5p}{6}=2.618}$$

b) the $y-axis$



Here the box is perpendicular to the axis of revolution so we use the Disk Method.

$R=2-y \quad r=y^2$

$$V=p\int_0^1\left((2-y)^2-(y^2)^2\right)dy=\boxed{\frac{32p}{15}=6.702}$$

c) $y=-2$

Shell Method:

$R=y+2 \quad h=2-y-y^2$

$$V=2p\int_0^1(y+2)(2-y-y^2)dy=\boxed{\frac{11p}{2}=17.288}$$

d) $x=4$

Disk Method

$R=4-y^2 \quad r=4-(2-y)$

$$V=p\int_0^1\left(\left(4-y^2\right)^2-\left(4-(2-y)\right)^2\right)dy=\boxed{\frac{36p}{5}}$$

#2: Let **R** be the region bounded by $x = 5 - y^2$ and $y = 3 - x$. Here we will interchange x and y and get $y = 5 - x^2$ and $x = 3 - y \quad \rightarrow \quad y = 3 - x$

Find the volume of the solids when **R** is revolved about:

$a)\quad x = 6 \quad \rightarrow \quad y = 6$

$b)\quad y = 4 \quad \rightarrow \quad x = 4$

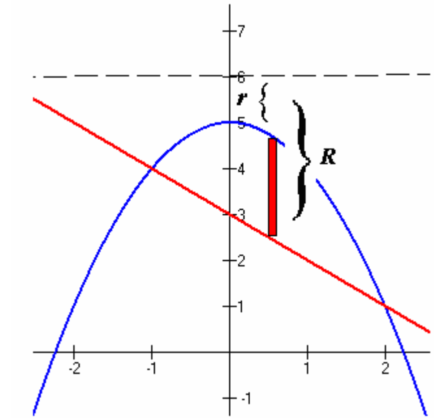
$a)\quad y = 6$

Disk Method:

$r = 6 - (5 - x^2) \quad R = 6 - (3 - x)$

$$V = p\int_{-1}^{2}\left((6-(3-x))^2 - (6-(5-x^2))^2\right)dx$$

$$= p\int_{-1}^{2}\left((6-y_2)^2 - (6-y_1)^2\right)dx = \boxed{\frac{117p}{5} = 73.513}$$



$b)\quad x = 4$

Shell Method

$R = 4 - x \quad h = (5 - x^2) - (3 - x)$

$$V = 2p\int_{-1}^{2}\left((4-x)((5-x^2)-(3-x)\right)dx$$

$$= 2p\int_{-1}^{2}[(4-x)(y_1 - y_2)]\,dx = \boxed{\frac{63p}{2} = 98.960}$$