Length of a Curve

I. Find the length of the given curve.

1. $f(x) = 3x^2 \quad -1 \le x \le 3$

$L = \int_{-1}^{3} \sqrt{1+(6x)^2}\,dx = \boxed{30.589}$

2. $f(x) = x^3 - x^2 + x \qquad 1 \le x \le 4$

$L = \int_{1}^{4} \sqrt{1+(3x^2-2x+1)^2}\,dx = \boxed{51.1697}$

3. $f(x) = 5\cos x \qquad 0 \le x \le p$

$L = \int_{0}^{p} \sqrt{1+(-5\sin x)^2}\,dx = \boxed{10.6969}$

4. $f(x) = \tan x \qquad -\frac{p}{3} \le x \le \frac{p}{3}$

$L = \int_{-\frac{p}{3}}^{\frac{p}{3}} \sqrt{1+(\sec^2 x)^2}\,dx = \boxed{4.114}$

5. $f(x) = \sin x - x\cos x \qquad 0 \le x \le p$

$L = \int_{0}^{p} \sqrt{1+(x\sin(x))^2}\,dx = \boxed{4.6984}$

II. Find the length of the boundary for the given region.

1. R is the region in the first quadrant bounded by $f(x) = 5\cos x$ and $g(x) = x + 2$.

The intersection of these two graphs is $a = -1.46324$ and $b = .941774$

$L_1 = \int_{a}^{b} \sqrt{1+(-5\sin x)^2}\,dx = 7.14098$

$L_2 = \int_{a}^{b} \sqrt{2}\,dx = 3.4012$

$L = 7.14098 + 3.4012 = \boxed{10.5422}$

2. R is the region bounded by $y = x^2, y = 1$ and $x = 2$.

$L_1 = \int_{1}^{2} \sqrt{1+(2x)^2}\,dx = 3.16784$

$L_2 = 1 \quad L_3 = 3$

$L = 3.16784 + 2 + 3 = \boxed{7.16784}$