Second Fundamental Theorem Problems

1. Let $G(x) = \int_0^x \sqrt{16 - t^2}\, dt$.

a) What is $G(0)$?

$G(0) = \int_0^0 \sqrt{16 - t^2}\, dt = \boxed{0}$

b) What is $G'(2)$?

$G'(x) = \sqrt{16 - x^2}$

$G'(2) = \sqrt{16 - 4} = \boxed{\sqrt{12}}$

2. Find $F'(x)$.

a) $F(x) = \int_0^x \frac{\sin t}{t}\, dt$

$\boxed{F'(x) = \frac{\sin x}{x}}$

b) $F(x) = \int_0^x e^{-t^2}\, dt$

$\boxed{F'(x) = e^{-x^2}}$

c) $F(x) = \int_1^{\cos x} \frac{1}{t}\, dt$

$\boxed{F'(x) = \frac{-\sin x}{\cos x} = -\tan x}$

d) $F(x) = \int_1^{x^2} \cos t\, dt$

$\boxed{F'(x) = 2x\cos(x^2)}$

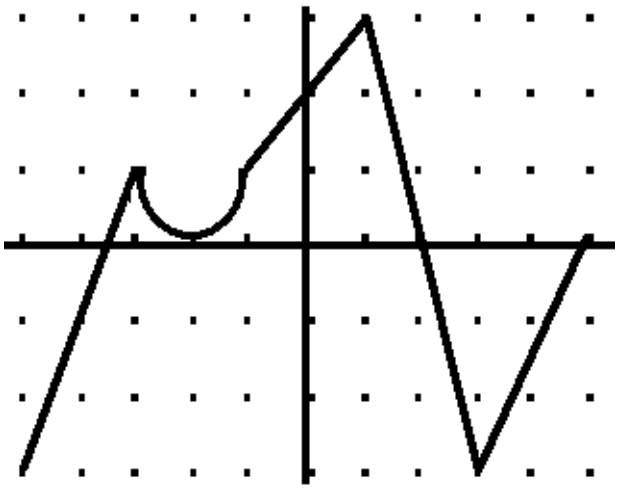
e) $F(x) = \int_4^{x^3} \frac{1}{2t}\, dt$

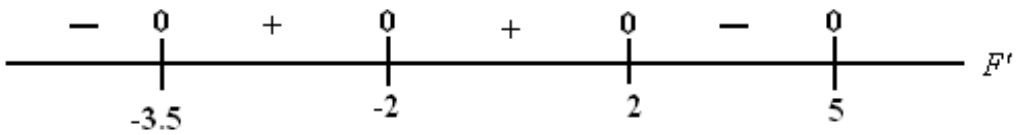
$\boxed{F'(x) = \frac{3x^2}{2x^3} = \frac{3}{2x}}$

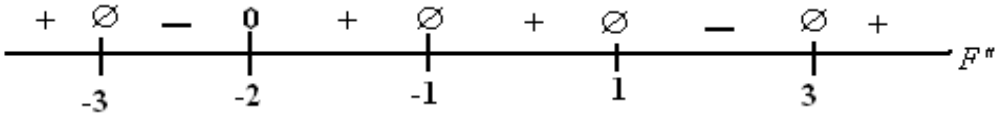
f) $F(x) = \int_0^1 e^{\tan t}\, dt$

$\boxed{F'(x) = 0}$

3. Let $F(x)=\int_0^x f(t)\,dt$ where f is the function graphed below.







a) Evaluate $F(2),\ F(0),\ and\ F(-1)$

$F(2)=\int_0^2 f(t)dt=\frac{5}{2}+\frac{3}{2}=\boxed{4}$

$F(0)=\int_0^0 f(t)dt=\boxed{0}$

$F(-1)=\int_0^{-1} f(t)dt=-\int_{-1}^0 f(t)dt=\boxed{-\frac{3}{2}}$

b) Find the *x*-coordinates of all relative maxima of F in the interval [-5,5].

There is a relative maximum at $x=2$ because F' goes + to −.

c) Identify all the inflection points of F in the interval [-5,5].

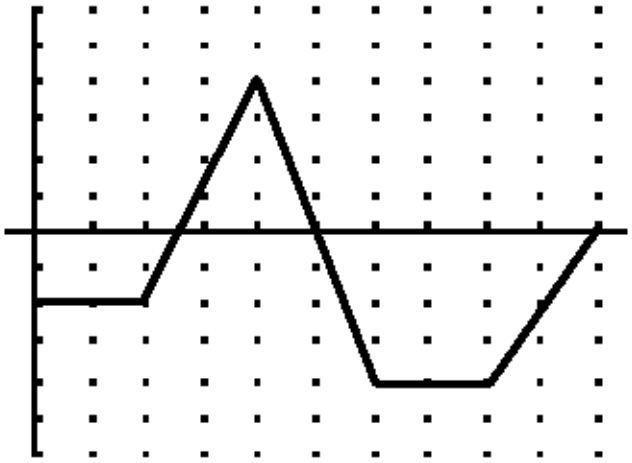
There are inflection points at $x=-3,-2,1,3$ because F'' changes sign.

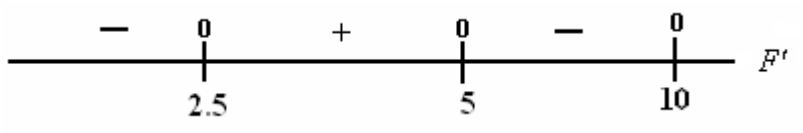
d) What is the average value of f on the interval [-5,5]?

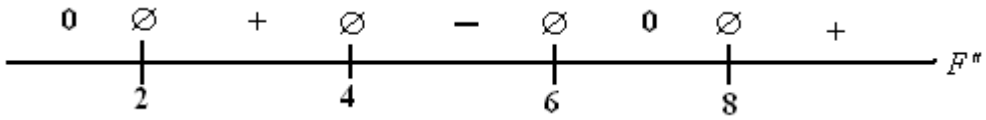
$\overline{f}=\frac{1}{10}\int_{-5}^5 f(t)dt=\frac{1}{10}\left[-\frac{9}{4}+\frac{1}{2}+\left(2-\frac{p}{2}\right)+\frac{3}{2}+\frac{5}{2}+\frac{3}{2}-\frac{9}{2}\right]$

$=\frac{1}{10}\left[\frac{5-2p}{4}\right]=\boxed{\frac{5-2p}{40}=-.032}$

4. Let $F(x)=\int_0^x f(t)\,dt$ where f is the function graphed below:







a) Evaluate $\int_0^2 f(t)dt,\ \int_0^4 f(t)dt,\ \int_2^4 f(t)dt,\ \int_5^{10} f(t)dt,\ \int_1^7 f(t)dt.$

$\int_0^2 f(t)dt = \boxed{-4}\quad \int_0^4 f(t)dt = \boxed{-1.5}\quad \int_2^4 f(t)dt = \boxed{2.5}\quad \int_5^{10} f(t)dt = \boxed{-14}\quad \int_1^7 f(t)dt = \boxed{-3.5}$

b) Evaluate $F(0),\ F(2),\ F(5)$ *and* $F(7).$

$F(0) = \int_0^0 f(t)dt = \boxed{0}$

$F(2) = \int_0^2 f(t)dt = \boxed{-4}$

$F(5) = \int_0^5 f(t)dt = \boxed{.5}$

$F(7) = \int_0^7 f(t)dt = \boxed{-5.5}$

c) On which subintervals of [0,10] is F decreasing?

F is decreasing when $0 < x < 2.5$ and $5 < x < 10$ because F' is negative.

d) On which subintervals of [0,10] is F increasing?

F is increasing when $2.5 < x < 5$ because F' is positive.

e) On which subintervals of [0,10] is F concave upward?

F is concave upward when $2 < x < 4$ and $8 < x < 10$ because F'' is positive.