Determine if the following functions are continuous at $x = 2$.

1. $f(x) = \begin{cases} 3x - 1 & x < 2 \\ x^2 + 1 & x \geq 2 \end{cases}$

2. $f(x) = \begin{cases} 6x - 5 & x < 2 \\ x + 3 & x \geq 2 \end{cases}$

3. $f(x) = \begin{cases} 4x + 1 & x < 2 \\ 3x^2 - 3 & x > 2 \end{cases}$

4. $f(x) = \begin{cases} \frac{1}{2}x + 1 & x > 2 \\ 3 - x & x \leq 2 \end{cases}$

5. $f(x) = \begin{cases} -2x & x \leq 2 \\ x^2 - 4x + 1 & x > 2 \end{cases}$

Answers

1. $f(x) = \begin{cases} 3x - 1 & x < 2 \\ x^2 + 1 & x \geq 2 \end{cases}$

$\lim_{x \to 2^-} f(x) = 5$ $\lim_{x \to 2^+} f(x) = 5$ $\lim_{x \to 2} f(x) = 5$ $f(2) = 5$

Thus, f is continuous at $x = 2$ because the limit equals the function.

2. $f(x) = \begin{cases} 6x - 5 & x < 2 \\ x + 3 & x \geq 2 \end{cases}$

$\lim_{x \to 2^-} f(x) = 7$ $\lim_{x \to 2^+} f(x) = 5$ $\lim_{x \to 2} f(x) = \varnothing$

Thus, f is not continuous at $x = 2$ because the limit does not exist.

3. $f(x) = \begin{cases} 4x + 1 & x < 2 \\ 3x^2 - 3 & x > 2 \end{cases}$

$\lim_{x \to 2^-} f(x) = 9$ $\lim_{x \to 2^+} f(x) = 9$ $\lim_{x \to 2} f(x) = 9$ $f(2) = \varnothing$

Thus, f is NOT continuous at $x = 2$ because the function is not defined at $x = 2$.

4. $f(x) = \begin{cases} \frac{1}{2}x + 1 & x > 2 \\ 3 - x & x \leq 2 \end{cases}$

$\lim_{x \to 2^-} f(x) = 1$ $\lim_{x \to 2^+} f(x) = 2$ $\lim_{x \to 2} f(x) = \varnothing$

Thus, f is not continuous at $x = 2$ because the limit does not exist.

5. $f(x) = \begin{cases} -2x & x \leq 2 \\ x^2 - 4x + 1 & x > 2 \end{cases}$

$\lim_{x \to 2^-} f(x) = -4$ $\lim_{x \to 2^+} f(x) = -3$ $\lim_{x \to 2} f(x) = \varnothing$

Thus, f is not continuous at $x = 2$ because the limit does not exist.